Lecture 1 (14/11/2024):

Matrices (Definition and Types of Matrices)

Introduction

Matrices are fundamental mathematical tools widely used in various fields such as mathematics, physics, engineering, computer science, and economics. A matrix is a rectangular array of numbers, symbols, or expressions arranged in rows and columns. This lecture will cover the definition of matrices and explore different types of matrices with examples.

Definition of a Matrix

A matrix is defined as a two-dimensional arrangement of elements, where the elements are placed in rows and columns. A matrix is typically denoted by a capital letter, and its elements are represented within brackets. For example:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2j} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3j} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ a_{\mathrm{ii}} & a_{\mathrm{i2}} & a_{\mathrm{i3}} & \cdots & a_{\mathrm{ij}} & \cdots & a_{\mathrm{in}} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ a_{\mathrm{ml}} & a_{\mathrm{m2}} & a_{\mathrm{m3}} & \cdots & a_{\mathrm{m}} & \cdots & a_{\mathrm{mn}} \end{bmatrix}$$

j-th column , i-th row

- mxn is the order (or size or dimension or degree) of the matrix · The number which appears at the intersection of the i-th row and j-th column is usually referred as the (i,j)-th entry of the matrix A and denoted by a_{ij} and the matrix A denoted by $[a_{\mathrm{ij}}]_{\mathrm{m\times n}}$ or$\mathcal{A}_{\mathrm{m\times n}}$.

* The entries α_{ij} of a matrix A may be real or complex (or any field) .

* If all entries of the matrix are real, then the matrix is called real matrix.

*If all entries of the matrix are complex, then the matrix is called complex matrix.

Types of Matrices

1. Row Matrix

A row matrix is a matrix that has only one row.

B = [2 5 7]

Here, matrix B is a 1×3 matrix.

2. Column Matrix

A column matrix is a matrix that has only one column.

$C = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ Here, matrix C is a 3×1 matrix.

3. Square Matrix

A square matrix is a matrix with the same number of rows and columns.

$$D = \begin{bmatrix} 1 & 1 & 2 \\ 4 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

Here, matrix D is a 3×3 square matrix.

4. Rectangular Matrix

A rectangular matrix is a matrix in which the number of rows is NOT equal to the number of columns.

$$D = \begin{bmatrix} 1 & 1 & 2 \\ \\ 0 & 2 & 1 \end{bmatrix}$$

matrix D is a 2×3 square matrix.

5. Diagonal Matrix

A diagonal matrix is a square matrix in which all elements except the diagonal elements are zero.

E = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Here, matrix E is a 3×3 diagonal matrix.

6. Identity Matrix

An identity matrix is a diagonal matrix where all diagonal elements are 1.

I = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Here, matrix I is a 3×3 identity matrix.

7. Zero Matrix

A zero matrix is a matrix in which all elements are zero.

For example:

$Z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Here, matrix Z is a 2×3 zero matrix.

8- Transpose of Matrix

The matrix resulting from replacing the rows of the matrix by the column of it and denoted by A' or A^T or A^t .

Example : If A=$\begin{bmatrix} 2 & 0 & 3 & -1 \\ 4 & 1 & 5 & 7 \end{bmatrix}_{2\times4}$ then A^t=$\begin{vmatrix} 2 & 4 \\ 0 & 1 \\ 3 & 5 \\ -1 & 7 \end{vmatrix}$.

9. Symmetric Matrix

A symmetric matrix is a square matrix that is equal to its transpose. $A = A^t$

For example:

A $= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ and $A^t = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ so A is symmetric matrix since $A = A^t$.

And $S = \begin{bmatrix} 6 & 3 & 2 \\ 3 & 0 & 5 \\ 2 & 5 & -4 \end{bmatrix}$

Here, matrix S is a 3×3 symmetric matrix.

10. skew Symmetric Matrix

is a square whose transpose equals its negative. That is, it satisfies the condition

$A = -A^t$.

The matrix

$$A = \begin{bmatrix} 0 & 2 & -45 \\ -2 & 0 & -4 \\ 45 & 4 & 0 \end{bmatrix}$$

$$-A = \begin{bmatrix} 0 & -2 & 45 \\ 2 & 0 & 4 \\ -45 & -4 & 0 \end{bmatrix} = A^{T}.$$

is skew-symmetric because $A = -A^t$.

11. Triangular Matrix

It is two kinds:

(*a*) Lower Triangular Matrix: A square matrix all its elements above the main diagonal are zeros, that $a_{ij} = 0$ for each $j > i$.

$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$ square lower triangular matrix of order 3∗3.

(b) Upper Triangular Matrix: A square matrix all its elements below the main diagonal are zeros, that is $a_{ij} = 0$ for each $i > j$.

$U = \begin{bmatrix} 1 & 2 & 5 \\ 0 & 2 & 5 \\ 0 & 0 & 1 \end{bmatrix}$ square upper triangular matrix of order $3*3$.

10. Scalar Matrix

A diagonal matrix is called scalar matrix if all main diagonal elements are equal.

$$S = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

11. Magic matrix

a square containing a number of integers arranged so that the sum of the numbers is the same in each row, column, and main diagonal and often in some or all of the other diagonals.

$M = \begin{bmatrix} 4 & 9 & 2 \\ 3 & 5 & 7 \\ 8 & 1 & 6 \end{bmatrix}$, 4+9+2=15 , 4+3+8=15, 4+5+6=15 , 2+7+6=15….

$$M= \begin{bmatrix} 6 & 3 & 10 & 15 \\ 9 & 16 & 5 & 4 \\ 7 & 2 & 11 & 14 \\ 12 & 13 & 8 & 1 \end{bmatrix}$$

Conclusion

Understanding the different types of matrices is essential for solving problems in linear algebra and its applications. Each type of matrix has unique properties that make it suitable for specific operations and applications.